Title: Meet a Mathematician

Brief Overview:

These activities will introduce students to Pascal's Triangle through the use of patterns in mathematics. Students will apply their knowledge of odd and even numbers, functions, triangular numbers, and other number relationships to analyze Pascal's Triangle.

Links to Standards:

- **Mathematics as Problem Solving**
  Students will demonstrate their ability to solve problems in mathematics including problems with open-ended answers, problems which are solved in a cooperative atmosphere, and problems which are solved with the use of technology.

- **Mathematics as Communication**
  Students will demonstrate their ability to communicate mathematically. They will read, write, and discuss mathematics with language and the signs, symbols, and terms of the discipline.

- **Mathematics as Reasoning**
  Students will demonstrate their ability to reason mathematically. They will make conjectures, gather evidence, and build arguments.

- **Mathematical Connections**
  Students will demonstrate their ability to connect mathematics topics within the discipline and with other disciplines.

- **Number Sense & Operations**
  Students will demonstrate their ability to describe and apply number relationships using concrete and abstract materials. They will choose appropriate operations and describe effects of operations on numbers.

- **Geometry & Spatial Sense**
  Students will demonstrate their ability to describe and apply geometric relationships using one, two, and three dimensional objects. They will demonstrate congruency, similarity, symmetry, and reflections and apply these concepts to the solution of geometric problems.

- **Patterns & Relationships**
  Students will demonstrate their ability to recognize numeric and geometric relationships and will generalize a relationship from data.

Grade/Level:

Grades 4-5

Duration/Length:

This lesson will take four class periods. Teacher discretion on duration based on level of group.

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Odd and even numbers
- Basic operation of a calculator
- How to complete a table

Objectives:

Students will:

- use the constant arithmetic feature on a calculator.
- identify patterns by using the constant arithmetic feature on a calculator.
- recognize and identify rules for functions.
- apply their knowledge of functions to problem solving situations.
- apply their knowledge of functions and patterns to find number relationships in Pascal's Triangle.

Materials/Resources/Printed Materials:

- calculator (with a constant arithmetic feature)
- student worksheets
- teacher resources
- pencils
- manipulatives (beans, pattern blocks)

Development/Procedures:

Task 1: Calculating Patterns
Students will identify patterns by using the constant arithmetic feature on a calculator.

Connections: calculator skills

- Provide each student with a calculator.
- Refer to Teacher Resource 1.
- Have students complete Student Resource 1.
- Once students have completed Student Resource 1, have them complete the following question in their math journals:

  Choose one of the tables from your Constant Arithmetic Feature sheet. What mathematical rule does your table show? Explain your mathematical rule. How do you know your table fits your rule?
- Teachers should evaluate journal entries based on the following criteria:

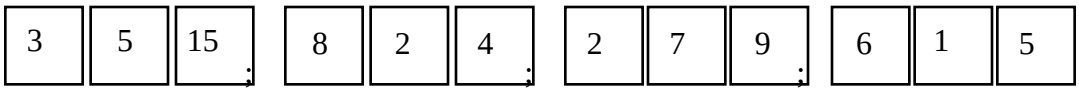
  - Students have used mathematics vocabulary.
  - Students have answered the question clearly and accurately.
  - Students have identified the table they chose.
  - Students have explained the mathematics rule which their table shows.
  - Students have explained <u>how</u> their table fits the rule.

- Hold a class discussion to share journal entries.
- Teacher Resource 2 provides the answers to Student Resource 1.

Task 2: What's My Function?
Students will recognize and identify rules for functions

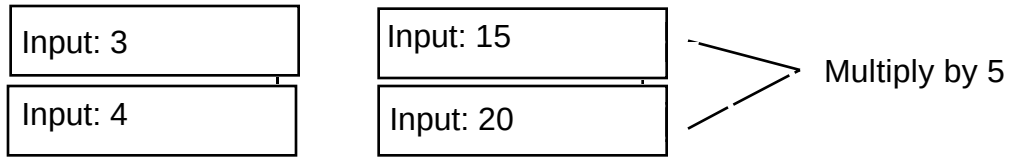
Connections: application of arithmetic operations.
numeration and number sense

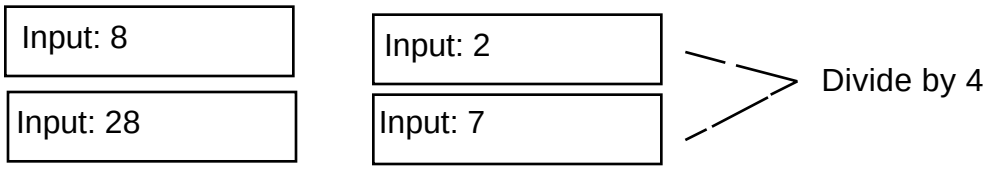
- Explain to students that we will be working with a function machine.
- Give students examples of function machines. For example:



(Have students determine the operation used in each example.)

- Introduce the first number as input the third number as output, have students find the rule.





- Create a table of input and output numbers. Have students complete the table and find the rule. (This could be done on the overhead, chalkboard, chart paper, file folders, learning stations, index cards.)
- Create a function machine to have in your classroom.

Directions: Choose a container for your machine.
Make two openings in your machine (one for input, one for output).
Label the openings.
Write the rule for your machine clearly on the container (ex. + 4).
(If a 3 is dropped in the input opening, a 7 would be the output.)

Resources**:**

Teaching Children Mathematics. February, 1997. Volume 3, No. 6.

Coburn, Terrence. Standards for School Mathematics Addenda Series, Grades K-6. Reston, National Council of Teachers of Mathematics, Inc. 1993.

Brisby, Linda Sue. Patterns and Functions. Solvang, CA. Hands On Inc.1990.

Task 3: Growing Patterns
Students will apply their knowledge of functions to problem solving situations.

Connections: functions

- Distribute Student Resource 2.
- Direct class's attention to the addition pattern which is formed with the triangular numbers. (The numbers 3, 6, and 10 are said to be triangular numbers because sets of 3, 6, or 10 members can be arranged in triangle.)
- Have students complete Student Resource 2 using manipulatives.
- Hold a class discussion to share results.

Note to Teacher: Triangles, rhombi, and parallelograms also work for triangular numbers. Teacher Resource 4 is a chart which may be used in conjunction with the extension on Student Resource 2.

Resources:

Henderson West, Beverly. The Prentice-Hall Encyclopedia of Mathematics. New Jersey, Prentice-Hall, Inc., 1982.

Charles, Randall. Problem-Solving Experiences in Mathematics. Menlo Park, California. Addison-Wesley, 1985.

Task 4: Pascal's Triangle
Students will apply their knowledge of functions and patterns to find number relationships in Pascal's Triangle.

Connections: functions
triangular numbers

- Provide students with information about the mathematician, Pascal. (See Teacher Resource Sheet 5)
- Display Pascal's Triangle (Student Resource 3) for the entire class. Explain to students that you have broken up the triangle into groups of numbers. Their job is to determine the relationship between their group's numbers. Once the relationship has been determined, students must independently state their relationship in their math journal. Here are some examples of groups of numbers from Pascal's Triangle:
(Note: The relationships for each group are shown in parenthesis. There may be more than one relationship between numbers.)

Group L: 1, 7, 21, 35, 35, 21, 7, 1.
(The sequence of numbers increase from 1-35, the exact sequence of numbers then decrease from 35-1.)

Group M: 1, 3, 6, 10, 15, 21, 28, 36, 45.
(The numbers are increasing first by 2, then 3, then 4, etc.)

Group N: 1, 7, 28, 84, 210.
(After the number 1, all of the numbers are factors of 7.)

Group O: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
(The numbers are counting numbers.)

Group P:

6 1
21 7 1
56 28 8 1

(Each number is the sum of the two numbers on either side, on the line above.
Ex.: 6 + 1 = 7 , 21 + 7 = 28)

- Break students into cooperative groups (no larger than four in a group).
- Hand out an index card to each group with a group of numbers from the triangle.
- (Be sure to put away Pascal's Triangle while students are working in their groups.)
- Hand out calculators to each student.
- Allow students time to work in groups.
- Have each group share their findings.
- Distribute copies of Pascal's Triangle (Student Resource 3) ask students to find patterns which have not been mentioned.
- Post a chart to display the relationships which were found in Pascal's Triangle. Encourage students to continue to look for patterns to add to the chart.

Example:

Pascal's Triangle The Source of Many Patterns -Counting Numbers

Teacher Note: Pascal's Triangle also has a relationship involving binary numbers.

$$1 = 1$$
$$1 + 1 = 2$$
$$1 + 2 + 1 = 4$$
$$1 + 3 + 3 + 1 = 8$$

1, 2, 4, and 8 are all binary numbers. This pattern of binary numbers continues down the triangle.

Sources:

Charles, Randall. Math*a*pedia. Menlow Park, California. Addison-Wesley, 1995. p. 75.

Smoothey, Marion. Let's Investigate Number Patterns. New York, Marshall Cavendish Corporation, 1993. p. 41-45.

Performance Assessment:

Assessment of student progress is ongoing throughout this unit. Included are reflection questions for the students to respond to as they complete each task. Teachers are encouraged to use observations and anecdotal records to assess students as they complete the tasks outlined in the learning unit. Teachers may also work with students to generate criteria to evaluate tasks.

Follow Up:

A good follow-up activity is to have students write a journal entry titled:

Meet a Mathematician: (Student's Name)

In this journal entry teachers should encourage students to reflect about the math they use everyday, their feelings about mathematics, and how mathematics impacts their lives.

Extension:

1. Have students research the following mathematicians:
   a. Leonardo of Pisa (thirteenth century) - Fibonacci sequence.
   b. Carl Gauss (1777-1855)
      -found sum of the first 100 natural numbers
      -greatest mathematician of 19th century
      -made contributions to astronomy and electricity
   c. George Polya (1887-1985) - Problem solving.
   -How to Solve It, printed 1945

2. Contact the National Security Agency Mathematics Speakers Bureau, to receive a catalog of topics for guest speakers in the area of mathematics.

   National Security Agency
   Mathematics Speakers Bureau
   ATTN: D5/MEPP
   Ft. George G. Meade, MD 20755-6217

3. Conduct "Patterns, Problem Solving, and Practice" Activities Integrating Math and Science (AIMS) activity.

Wiebe, Arthur. "Patterns, Problem Solving, and Practice". AIMS, March 1996. Volume X, Number 8. p. 18-23.

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Using the Constant Arithmetic Feature

1. Provide each student with a calculator.
2. Ask the students to enter 0 + 2 = into their calculators.
   (*Note to instructor: 2 is the constant number in both examples.*)
3. Teacher instructs students as a class to depress the = key once...twice...three times...four times. Display sentence strip:

0+2=,=,=,=

4. Have a class discussion of the following questions:
   - -What number did you end up with after you hit the = key four times?
   - -What happened to the numbers on the calculator as you continued to hit the = key? How do you know?
5. Repeat steps 2-4 having students use the number sentence 1+2=, be sure to mention to the students that your .

1+2=,=,=,=

6. Using Student Response Sheet 1, have students repeat the exercise recording their data on a table (Student Resource 1).

Using the Constant Arithmetic Feature

(The constant arithmetic feature is the = sign.)

Directions: Complete the tables using the constant arithmetic feature. (Be sure to fill in the table every time you depress the = sign.)

0 + 2 =

Number of times = is entered	1	2						
Display	2							

1 + 2 =

Number of times = is entered	1	2						
Display	3							

Using the Constant Arithmetic Feature

(The constant arithmetic feature is the = sign.)

Directions: Complete the tables using the constant arithmetic feature. (Be sure to fill in the table every time you depress the = sign.)

0 + 2 =

Number of times = is entered	1	2	3	4	5	6	7	8
Display	2	4	6	8	10	12	14	16

1 + 2 =

Number of times = is entered	1	2	3	4	5	6	7	8
Display	3	5	7	9	11	13	15	17

Meet a Mathematician
Student Resource 2

Directions: Read the problem below. Use manipulatives to help you solve the problem. Fill in the table.

Jill's Beans

One night, Jill had lima beans for dinner. Jill hates lima beans, so instead of eating them she used them to make shapes on her plate. Jill arranged her beans into a triangle. Her triangle had two rows and had used a total of three beans. She continued making her triangle larger by adding one row at a time. Each new row had one bean more than the row above it. Jill was finished when she had used all forty-five of her beans. How many rows of beans were in Jill's triangle?

(Hint: Look for a pattern. Then complete the pattern.)

1 1+2=3 1+2+3=6

Number of rows in the triangle	2								
Total number of beans in the triangle	3								

Jill was sent to her room for not eating her beans. She was not allowed to use the phone, watch TV, play video games, or listen to the radio. She was **SO** frustrated that she knocked over her container of pattern blocks. As she was picking them up, she looked at the triangles and said, "I wonder if I can make triangles of different sizes using just triangles."

(Was Jill able to make triangles of different sizes using just triangles? Compare your findings with the table.)

Extension: Will this same pattern work with other pattern block shapes? Why or why not? Explain your answer and give a rule.

Adapted from: Writing Mathematics : Grade 5. Creative Publications, 1995.

Pascal's Triangle

Many patterns can be found in Pascal's Triangle, which extends downward forever. A portion of the triangle is pictured below.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

1 10 45 120 210 252 210 120 45 10 1

1 11 55 165 330 462 462 330 165 55 11 1

1 12 66 225 495 792 924 792 495 225 66 12 1

Directions: Read the problem below. Use manipulatives to help you solve the problem. Fill in the table.

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(Hint: Look for a pattern. Then complete the pattern.)

1 1+2=3 1+2+3=6

Number of rows in the triangle	2	3	4	5	6	7	8	9		
Total number of beans in the triangle	3	6	10	15	21	28	36	45		

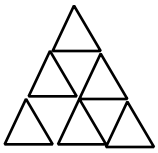
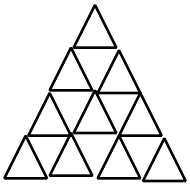
Jill was sent to here room for not eating her beans. She was not allowed to use the phone, watch TV, play video games, or listen to the radio. She was **SO** frustrated that she knocked over her container of pattern blocks. As she was picking them up, she looked at the triangles and said, "I wonder if I can make triangles of different sizes using just triangles."

(Was Jill able to make triangles of different sizes using just triangles? Compare your findings with the table.)

Extension: Will this same pattern work with other pattern block shapes? Why or why not? Explain your answer and give a rule.

Adapted from: Writing Mathematics : Grade 5. Creative Publications, 1995.

Charting Growing Patterns

Model (will vary according to shape)	Number of Blocks Needed
	1
	4
	9
	16

Adapted from: Writing Mathematics : Grade 5. Creative Publications, 1995.

Blaise Pascal

Blaise Pascal was a famous mathematician who lived in France in the 1600's. Even as a child Pascal was fascinated by mathematics. In fact, he even created the first calculator that worked. Pascal was very interested in probability (the study of chance). He consulted several gamblers of his time to help him in his study of probability. Pascal's studies led him to a triangular pattern of numbers, which is now known as Pascal's Triangle. Pascal's Triangle is used to determine the probability of combinations of numbers.

Sources:
Math*a*pedia. Addison Wesley, 1995.
Let's Investigate Number Patterns. Marshall Cavendish Corporation, 1993.